Some interesting characteristics of quotients of conjugates :

First of all, remember that $\overline{\boldsymbol{a} + \boldsymbol{bi}}$ is the notation for the conjugate of $\boldsymbol{a} + \boldsymbol{bi}$;
that is, $\overline{\boldsymbol{a} + \boldsymbol{bi}} = \boldsymbol{a} - \boldsymbol{bi}$ and $\overline{\boldsymbol{a} - \boldsymbol{bi}} = \boldsymbol{a} + \boldsymbol{bi}$

Comparing Two Quotients :

Case 1 :

$$\frac{\text{Bicomplex \#1}}{\text{Bicomplex \#2}} \quad \text{vs.} \quad \frac{\overline{\text{Bicomplex \#1}}}{\overline{\text{Bicomplex \#2}}} \; ; \text{ that is,}$$

$$\frac{\boldsymbol{a} + \boldsymbol{bi}}{\boldsymbol{c} - \boldsymbol{di}} \quad \text{vs.} \quad \frac{\boldsymbol{a} - \boldsymbol{bi}}{\boldsymbol{c} + \boldsymbol{di}}$$

Think about what is necessary to rationalize a quotient involving a bicomplex divisor…
The conjugate, correct?

$$\frac{\boldsymbol{a} + \boldsymbol{bi}}{\boldsymbol{c} - \boldsymbol{di}} = \left(\frac{\boldsymbol{a} + \boldsymbol{bi}}{\boldsymbol{c} - \boldsymbol{di}}\right)\left(\frac{\boldsymbol{c} + \boldsymbol{di}}{\boldsymbol{c} + \boldsymbol{di}}\right) = \frac{(\boldsymbol{ac} - \boldsymbol{bd}) + (\boldsymbol{ad} + \boldsymbol{bc})\boldsymbol{i}}{\boldsymbol{c}^2 + \boldsymbol{d}^2} = \left(\frac{\boldsymbol{ac} - \boldsymbol{bd}}{\boldsymbol{c}^2 + \boldsymbol{d}^2}\right) + \left(\frac{\boldsymbol{ad} + \boldsymbol{bc}}{\boldsymbol{c}^2 + \boldsymbol{d}^2}\right)\boldsymbol{i}$$

vs.

$$\frac{\boldsymbol{a} - \boldsymbol{bi}}{\boldsymbol{c} + \boldsymbol{di}} = \left(\frac{\boldsymbol{a} - \boldsymbol{bi}}{\boldsymbol{c} + \boldsymbol{di}}\right)\left(\frac{\boldsymbol{c} - \boldsymbol{di}}{\boldsymbol{c} - \boldsymbol{di}}\right) = \frac{(\boldsymbol{ac} - \boldsymbol{bd}) - (\boldsymbol{ad} + \boldsymbol{bc})\boldsymbol{i}}{\boldsymbol{c}^2 + \boldsymbol{d}^2} = \left(\frac{\boldsymbol{ac} - \boldsymbol{bd}}{\boldsymbol{c}^2 + \boldsymbol{d}^2}\right) - \left(\frac{\boldsymbol{ad} + \boldsymbol{bc}}{\boldsymbol{c}^2 + \boldsymbol{d}^2}\right)\boldsymbol{i}$$

WHICH ARE CONJUGATES!

Now let’s get radical, shall we ?

Case 2 :

$$\frac{\text{Biradical \#1}}{\text{Biradical \#2}} \text{ vs. } \frac{\overline{\text{Biradical \#1}}}{\overline{\text{Biradical \#2}}} \text{ ; that is,}$$

$$\frac{a + \sqrt{b}}{c - \sqrt{d}} \text{ vs. } \frac{a - \sqrt{b}}{c + \sqrt{d}}$$

Think about what is necessary to rationalize a quotient involving a biradical divisor…
The conjugate again, correct?

$$\frac{a + \sqrt{b}}{c - \sqrt{d}} = \left(\frac{a + \sqrt{b}}{c - \sqrt{d}}\right)\left(\frac{c + \sqrt{d}}{c + \sqrt{d}}\right) = \frac{\left(ac - \sqrt{bd}\right) + \left(a\sqrt{d} + c\sqrt{b}\right)}{c^2 - d} = \left(\frac{ac - \sqrt{bd}}{c^2 - d}\right) + \left(\frac{a\sqrt{d} + c\sqrt{b}}{c^2 - d}\right)$$

vs.

$$\frac{a - \sqrt{b}}{c + \sqrt{d}} = \left(\frac{a - \sqrt{b}}{c + \sqrt{d}}\right)\left(\frac{c - \sqrt{d}}{c - \sqrt{d}}\right) = \frac{\left(ac - \sqrt{bd}\right) - \left(a\sqrt{d} + c\sqrt{b}\right)}{c^2 - d} = \left(\frac{ac - \sqrt{bd}}{c^2 - d}\right) - \left(\frac{a\sqrt{d} + c\sqrt{b}}{c^2 - d}\right)$$

WHICH ARE AGAIN CONJUGATES!

How about we get complex and radical?

Case 3 :

$$\frac{\text{Bicomrad \#1}}{\text{Bicomrad \#2}} \quad \text{vs.} \quad \frac{\overline{\text{Bicomrad \#1}}}{\overline{\text{Bicomrad \#2}}} \quad ; \text{ that is,}$$

$$\frac{a + i\sqrt{b}}{c - i\sqrt{d}} \quad \text{vs.} \quad \frac{a - i\sqrt{b}}{c + i\sqrt{d}}$$

Think about what is necessary to rationalize a quotient involving a bicomrad divisor…
The conjugate again, correct?

$$\begin{aligned}
&\frac{a + i\sqrt{b}}{c - i\sqrt{d}} \\
&= \left(\frac{a + i\sqrt{b}}{c - i\sqrt{d}}\right)\left(\frac{c + i\sqrt{d}}{c + i\sqrt{d}}\right) \\
&= \frac{\left(ac - \sqrt{bd}\right) + \left(a\sqrt{d} + c\sqrt{b}\right)i}{c^2 + d} \\
&= \left(\frac{ac - \sqrt{bd}}{c^2 + d}\right) + \left(\frac{a\sqrt{d} + c\sqrt{b}}{c^2 + d}\right)i
\end{aligned}$$

vs.

$$\begin{aligned}
&\frac{a - i\sqrt{b}}{c + i\sqrt{d}} \\
&= \left(\frac{a - i\sqrt{b}}{c + i\sqrt{d}}\right)\left(\frac{c - i\sqrt{d}}{c - i\sqrt{d}}\right) \\
&= \frac{\left(ac - \sqrt{bd}\right) - \left(a\sqrt{d} + c\sqrt{b}\right)i}{c^2 + d} \\
&= \left(\frac{ac - \sqrt{bd}}{c^2 + d}\right) - \left(\frac{a\sqrt{d} + c\sqrt{b}}{c^2 + d}\right)i
\end{aligned}$$

WHAT DO YOU… CONJUGATES AGAIN!

I know what you’re thinking … Does that work with binomials too?

Case 4 :

$$\frac{\text{Binomial \#1}}{\text{Binomial \#2}} \text{ vs. } \frac{\overline{\text{Binomial \#1}}}{\overline{\text{Binomial \#2}}} \text{ ; that is,}$$

$$\frac{\boldsymbol{a} + \boldsymbol{b}}{\boldsymbol{c} - \boldsymbol{d}} \text{ vs. } \frac{\boldsymbol{a} - \boldsymbol{b}}{\boldsymbol{c} + \boldsymbol{d}}$$

Think about what is necessary to compare these two quotients…
You'd need to establish a common denominator, which would once again require that you employ the power of the conjugate, correct?

$$\begin{aligned} &\frac{\boldsymbol{a} + \boldsymbol{b}}{\boldsymbol{c} - \boldsymbol{d}} \\ &= \left(\frac{\boldsymbol{a} + \boldsymbol{b}}{\boldsymbol{c} - \boldsymbol{d}}\right)\left(\frac{\boldsymbol{c} + \boldsymbol{d}}{\boldsymbol{c} + \boldsymbol{d}}\right) \\ &= \frac{(\boldsymbol{ac} + \boldsymbol{bd}) + (\boldsymbol{ad} + \boldsymbol{bc})}{\boldsymbol{c}^2 - \boldsymbol{d}^2} \\ &= \left(\frac{\boldsymbol{ac} + \boldsymbol{bd}}{\boldsymbol{c}^2 - \boldsymbol{d}^2}\right) + \left(\frac{\boldsymbol{ad} + \boldsymbol{bc}}{\boldsymbol{c}^2 - \boldsymbol{d}^2}\right) \end{aligned} \quad \text{vs.} \quad \begin{aligned} &\frac{\boldsymbol{a} - \boldsymbol{b}}{\boldsymbol{c} + \boldsymbol{d}} \\ &= \left(\frac{\boldsymbol{a} - \boldsymbol{b}}{\boldsymbol{c} + \boldsymbol{d}}\right)\left(\frac{\boldsymbol{c} - \boldsymbol{d}}{\boldsymbol{c} - \boldsymbol{d}}\right) \\ &= \frac{(\boldsymbol{ac} + \boldsymbol{bd}) - (\boldsymbol{ad} + \boldsymbol{bc})}{\boldsymbol{c}^2 - \boldsymbol{d}^2} \\ &= \left(\frac{\boldsymbol{ac} + \boldsymbol{bd}}{\boldsymbol{c}^2 - \boldsymbol{d}^2}\right) - \left(\frac{\boldsymbol{ad} + \boldsymbol{bc}}{\boldsymbol{c}^2 - \boldsymbol{d}^2}\right) \end{aligned}$$

AHA… CONJUGATES!

So are you thinking we need to get rational here, now?

Case 5 :

$$\frac{\text{Birational \#1}}{\text{Birational \#2}} \text{ vs. } \frac{\overline{\text{Birational \#1}}}{\overline{\overline{\text{Birational \#2}}}} \text{ ; that is,}$$

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{e}{f}-\frac{g}{h}} \text{ vs. } \frac{\frac{a}{b}-\frac{c}{d}}{\frac{e}{f}+\frac{g}{h}}$$

Think about what is necessary to compare these two quotients, which are complex fractions… Using Method 2 for clearing a complex fraction of its internal fractions, the Multiplicative Identity is employed. That is, you need to multiply the numerator and the denominator of the complex fraction by the LCD $(bdfh)$ of all the little fractions within the complex fraction, and at some point (see * & ** below) you will once again need to employ a conjugate to compare the results. Allow me to demonstrate…

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{e}{f}-\frac{g}{h}}$$

$$= \frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}+\frac{c}{d}}{\frac{e}{f}-\frac{g}{h}}\right)$$

$$= \frac{adfh + cbfh}{ebdh - gbdf}$$

$$= \frac{fh(ad+bc)}{bd(eh-gf)}$$

$$= \frac{fh(ad+bc)}{bd(eh-gf)}\left(\frac{eh+gf}{eh+gf}\right) \qquad *$$

$$= \frac{fh}{bd}\left[\frac{(adeh+bcgf)+(adgf+bceh)}{e^2h^2-g^2f^2}\right]$$

$$= \left(\frac{fh(adeh+bcgf)}{bd(e^2h^2-g^2f^2)}\right)+\left(\frac{fh(adgf+bceh)}{bd(e^2h^2-g^2f^2)}\right)$$

vs.

$$\frac{\frac{a}{b}-\frac{c}{d}}{\frac{e}{f}+\frac{g}{h}}$$

$$= \frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}-\frac{c}{d}}{\frac{e}{f}+\frac{g}{h}}\right)$$

$$= \frac{adfh - cbfh}{ebdh + gbdf}$$

$$= \frac{fh(ad-bc)}{bd(eh+gf)}$$

$$= \frac{fh(ad-bc)}{bd(eh+gf)}\left(\frac{eh-gf}{eh-gf}\right) \qquad **$$

$$= \frac{fh}{bd}\left[\frac{(adeh+bcgf)-(adgf+bceh)}{e^2h^2-g^2f^2}\right]$$

$$= \left(\frac{fh(adeh+bcgf)}{bd(e^2h^2-g^2f^2)}\right)-\left(\frac{fh(adgf+bceh)}{bd(e^2h^2-g^2f^2)}\right)$$

WHICH ARE, AMAZINGLY SO, AGAIN CONJUGATES !
Can we get similar results from quotients involving biradrats ?

Case 6 :

$$\frac{\text{Biradrat \#1}}{\text{Biradrat \#2}} \text{ vs. } \frac{\overline{\text{Biradrat \#1}}}{\overline{\text{Biradrat \#2}}}$$; that is,

$$\frac{\frac{a}{b}+\frac{\sqrt{c}}{d}}{\frac{e}{f}-\frac{\sqrt{g}}{h}} \text{ vs. } \frac{\frac{a}{b}-\frac{\sqrt{c}}{d}}{\frac{e}{f}+\frac{\sqrt{g}}{h}}$$

Once again, you need to multiply the numerator and the denominator of the complex fraction by the LCD $(bdfh)$ of all the little fractions within the complex fraction, and at some point (see * & ** below) you will once again need to employ a conjugate to compare the results. Allow me to demonstrate…

$$\frac{\frac{a}{b}+\frac{\sqrt{c}}{d}}{\frac{e}{f}-\frac{\sqrt{g}}{h}} \qquad\qquad \frac{\frac{a}{b}-\frac{\sqrt{c}}{d}}{\frac{e}{f}+\frac{\sqrt{g}}{h}}$$

$$=\frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}+\frac{\sqrt{c}}{d}}{\frac{e}{f}-\frac{\sqrt{g}}{h}}\right) \qquad\qquad =\frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}-\frac{\sqrt{c}}{d}}{\frac{e}{f}+\frac{\sqrt{g}}{h}}\right)$$

$$=\frac{adfh+bfh\sqrt{c}}{ebdh-bdf\sqrt{g}} \qquad\qquad =\frac{adfh-bfh\sqrt{c}}{ebdh+bdf\sqrt{g}}$$

$$=\frac{fh\left(ad+b\sqrt{c}\right)}{bd\left(eh-f\sqrt{g}\right)} \qquad\qquad =\frac{fh\left(ad-b\sqrt{c}\right)}{bd\left(eh+f\sqrt{g}\right)}$$

$$=\frac{fh\left(ad+b\sqrt{c}\right)}{bd\left(eh-f\sqrt{g}\right)}\left(\frac{eh+f\sqrt{g}}{eh+f\sqrt{g}}\right) \quad * \qquad\qquad =\frac{fh\left(ad-b\sqrt{c}\right)}{bd\left(eh+f\sqrt{g}\right)}\left(\frac{eh-f\sqrt{g}}{eh-f\sqrt{g}}\right) \quad **$$

$$=\frac{fh}{bd}\left[\frac{\left(adeh+bf\sqrt{cg}\right)+\left(adf\sqrt{g}+beh\sqrt{c}\right)}{e^2h^2-f^2g}\right] \qquad\qquad =\frac{fh}{bd}\left[\frac{\left(adeh+bf\sqrt{cg}\right)-\left(adf\sqrt{g}+beh\sqrt{c}\right)}{e^2h^2-f^2g}\right]$$

$$=\left(\frac{fh\left(adeh+bf\sqrt{cg}\right)}{bd(e^2h^2-f^2g)}\right)+\left(\frac{fh\left(adf\sqrt{g}+beh\sqrt{c}\right)}{bd(e^2h^2-f^2g)}\right) \text{ vs. } =\left(\frac{fh\left(adeh+bf\sqrt{cg}\right)}{bd(e^2h^2-f^2g)}\right)-\left(\frac{fh\left(adf\sqrt{g}+beh\sqrt{c}\right)}{bd(e^2h^2-f^2g)}\right)$$

AGAIN CONJUGATES ! … MAYBE THIS ISN'T SO AMAZING ANYMORE…

OK; last one … How about combining complexity, rationality, & radicality …

Allow me to coin one more "term" , bicomradrats , while dealing with quotients involving complexity, rationality, & radicality ?

Case 7 :

$$\frac{\text{Bicomradrat \#1}}{\text{Bicomradrat \#2}} \text{ vs. } \frac{\overline{\text{Bicomradrat \#1}}}{\overline{\text{Bicomradrat \#2}}} \text{ ; that is,}$$

$$\frac{\frac{a}{b}+\frac{i\sqrt{c}}{d}}{\frac{e}{f}-\frac{i\sqrt{g}}{h}} \text{ vs. } \frac{\frac{a}{b}-\frac{i\sqrt{c}}{d}}{\frac{e}{f}+\frac{i\sqrt{g}}{h}}$$

Once again, you need to multiply the numerator and the denominator of the complex fraction by the LCD $(bdfh)$ of all the little fractions within the complex fraction, and at some point (see * & ** below) you will once again need to employ a conjugate to compare the results.

$$\frac{\frac{a}{b}+\frac{i\sqrt{c}}{d}}{\frac{e}{f}-\frac{i\sqrt{g}}{h}}$$

$$= \frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}+\frac{i\sqrt{c}}{d}}{\frac{e}{f}-\frac{i\sqrt{g}}{h}}\right)$$

$$= \frac{adfh+(bfh\sqrt{c})i}{ebdh-(bdf\sqrt{g})i}$$

$$= \frac{fh\left(ad+bi\sqrt{c}\right)}{bd\left(eh-fi\sqrt{g}\right)}$$

$$= \frac{fh\left(ad+bi\sqrt{c}\right)}{bd\left(eh-fi\sqrt{g}\right)}\left(\frac{eh+fi\sqrt{g}}{eh+fi\sqrt{g}}\right) \quad *$$

$$= \frac{fh}{bd}\left[\frac{\left(adeh-bf\sqrt{cg}\right)+\left(adf\sqrt{g}+beh\sqrt{c}\right)i}{e^2h^2+f^2g}\right]$$

$$= \left(\frac{fh\left(adeh-bf\sqrt{cg}\right)}{bd(e^2h^2+f^2g)}\right)+\left(\frac{fh\left(adf\sqrt{g}+beh\sqrt{c}\right)i}{bd(e^2h^2+f^2g)}\right)$$

vs.

$$\frac{\frac{a}{b}-\frac{i\sqrt{c}}{d}}{\frac{e}{f}+\frac{i\sqrt{g}}{h}}$$

$$= \frac{bdfh}{bdfh}\left(\frac{\frac{a}{b}-\frac{i\sqrt{c}}{d}}{\frac{e}{f}+\frac{i\sqrt{g}}{h}}\right)$$

$$= \frac{adfh-(bfh\sqrt{c})i}{ebdh+(bdf\sqrt{g})i}$$

$$= \frac{fh\left(ad-bi\sqrt{c}\right)}{bd\left(eh+fi\sqrt{g}\right)}$$

$$= \frac{fh\left(ad-bi\sqrt{c}\right)}{bd\left(eh+fi\sqrt{g}\right)}\left(\frac{eh-fi\sqrt{g}}{eh-fi\sqrt{g}}\right) \quad **$$

$$= \frac{fh}{bd}\left[\frac{\left(adeh-bf\sqrt{cg}\right)-\left(adf\sqrt{g}+beh\sqrt{c}\right)i}{e^2h^2+f^2g}\right]$$

$$= \left(\frac{fh\left(adeh-bf\sqrt{cg}\right)}{bd(e^2h^2+f^2g)}\right)-\left(\frac{fh\left(adf\sqrt{g}+beh\sqrt{c}\right)i}{bd(e^2h^2+f^2g)}\right)$$

WOW CONJUGATES ! … I didn't see that one coming </sarcasm>
I'm out …