### NCTM Standards (2000) for Grades 3\(^{rd}\) through 5\(^{th}\)

#### Number and Operations Standard for Grades 3-5

<table>
<thead>
<tr>
<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
<th>In grades 3–5 all students should—</th>
</tr>
</thead>
</table>
| **Understand numbers, ways of representing numbers, relationships among numbers, and number systems** | • understand the place-value structure of the base-ten number system and be able to represent and compare whole numbers and decimals;  
• recognize equivalent representations for the same number and generate them by decomposing and composing numbers;  
• develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers;  
• use models, benchmarks, and equivalent forms to judge the size of fractions;  
• recognize and generate equivalent forms of commonly used fractions, decimals, and percents;  
• explore numbers less than 0 by extending the number line and through familiar applications;  
• describe classes of numbers according to characteristics such as the nature of their factors. |
| **Understand meanings of operations and how they relate to one another** | • understand various meanings of multiplication and division;  
• understand the effects of multiplying and dividing whole numbers;  
• identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems;  
• understand and use properties of operations, such as the distributivity of multiplication over addition. |
| **Compute fluently and make reasonable estimates** | • develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 × 50;  
• develop fluency in adding, subtracting, multiplying, and dividing whole numbers;  
• develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results. |
Algebra Standard for Grades 3-5

<table>
<thead>
<tr>
<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
<th>In grades 3–5 all students should—</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand patterns, relations, and functions</td>
<td>• describe, extend, and make generalizations about geometric and numeric patterns; • represent and analyze patterns and functions, using words, tables, and graphs.</td>
</tr>
<tr>
<td>Represent and analyze mathematical situations and structures using algebraic symbols</td>
<td>• identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers; • represent the idea of a variable as an unknown quantity using a letter or a symbol; • express mathematical relationships using equations.</td>
</tr>
<tr>
<td>Use mathematical models to represent and understand quantitative relationships</td>
<td>• model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.</td>
</tr>
<tr>
<td>Analyze change in various contexts</td>
<td>• investigate how a change in one variable relates to a change in a second variable; • identify and describe situations with constant or varying rates of change and compare them.</td>
</tr>
</tbody>
</table>
## Geometry Standard for Grades 3-5

<table>
<thead>
<tr>
<th>Expectations</th>
<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
<th>In grades 3–5 all students should—</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships</td>
<td>• identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;</td>
<td>• identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes;</td>
</tr>
<tr>
<td></td>
<td>• classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;</td>
<td>• classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids;</td>
</tr>
<tr>
<td></td>
<td>• investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;</td>
<td>• investigate, describe, and reason about the results of subdividing, combining, and transforming shapes;</td>
</tr>
<tr>
<td></td>
<td>• explore congruence and similarity;</td>
<td>• explore congruence and similarity;</td>
</tr>
<tr>
<td></td>
<td>• make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.</td>
<td>• make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.</td>
</tr>
<tr>
<td>Specify locations and describe spatial relationships using coordinate geometry and other representational systems</td>
<td>• describe location and movement using common language and geometric vocabulary;</td>
<td>• describe location and movement using common language and geometric vocabulary;</td>
</tr>
<tr>
<td></td>
<td>• make and use coordinate systems to specify locations and to describe paths;</td>
<td>• make and use coordinate systems to specify locations and to describe paths;</td>
</tr>
<tr>
<td></td>
<td>• find the distance between points along horizontal and vertical lines of a coordinate system.</td>
<td>• find the distance between points along horizontal and vertical lines of a coordinate system.</td>
</tr>
<tr>
<td>Apply transformations and use symmetry to analyze mathematical situations</td>
<td>• predict and describe the results of sliding, flipping, and turning two-dimensional shapes;</td>
<td>• predict and describe the results of sliding, flipping, and turning two-dimensional shapes;</td>
</tr>
<tr>
<td></td>
<td>• describe a motion or a series of motions that will show that two shapes are congruent;</td>
<td>• describe a motion or a series of motions that will show that two shapes are congruent;</td>
</tr>
<tr>
<td></td>
<td>• identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.</td>
<td>• identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.</td>
</tr>
<tr>
<td>Use visualization, spatial reasoning, and geometric modeling to solve problems</td>
<td>• build and draw geometric objects;</td>
<td>• build and draw geometric objects;</td>
</tr>
<tr>
<td></td>
<td>• create and describe mental images of objects, patterns, and paths;</td>
<td>• create and describe mental images of objects, patterns, and paths;</td>
</tr>
<tr>
<td></td>
<td>• identify and build a three-dimensional object from two-dimensional representations of that object;</td>
<td>• identify and build a three-dimensional object from two-dimensional representations of that object;</td>
</tr>
<tr>
<td></td>
<td>• identify and draw a two-dimensional representation of a three-dimensional object;</td>
<td>• identify and draw a two-dimensional representation of a three-dimensional object;</td>
</tr>
<tr>
<td></td>
<td>• use geometric models to solve problems in other areas of mathematics, such as number and measurement;</td>
<td>• use geometric models to solve problems in other areas of mathematics, such as number and measurement;</td>
</tr>
<tr>
<td></td>
<td>• recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.</td>
<td>• recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.</td>
</tr>
</tbody>
</table>
# Measurement Standard for Grades 3-5

**Instructional programs from prekindergarten through grade 12 should enable all students to—**

<table>
<thead>
<tr>
<th>Understand measurable attributes of objects and the units, systems, and processes of measurement</th>
<th>In grades 3–5 all students should—</th>
</tr>
</thead>
</table>
| • understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute;  
• understand the need for measuring with standard units and become familiar with standard units in the customary and metric systems;  
• carry out simple unit conversions, such as from centimeters to meters, within a system of measurement;  
• understand that measurements are approximations and how differences in units affect precision;  
• explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way. | |

| Apply appropriate techniques, tools, and formulas to determine measurements | • develop strategies for estimating the perimeters, areas, and volumes of irregular shapes;  
• select and apply appropriate standard units and tools to measure length, area, volume, weight, time, temperature, and the size of angles;  
• select and use benchmarks to estimate measurements;  
• develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms;  
• develop strategies to determine the surface areas and volumes of rectangular solids. |
# Data Analysis and Probability Standard for Grades 3-5

**Expectations**

<table>
<thead>
<tr>
<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
<th>In grades 3–5 all students should—</th>
</tr>
</thead>
</table>
| **Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them** | • design investigations to address a question and consider how data-collection methods affect the nature of the data set;  
• collect data using observations, surveys, and experiments;  
• represent data using tables and graphs such as line plots, bar graphs, and line graphs;  
• recognize the differences in representing categorical and numerical data. |
| **Select and use appropriate statistical methods to analyze data** | • describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed;  
• use measures of center, focusing on the median, and understand what each does and does not indicate about the data set;  
• compare different representations of the same data and evaluate how well each representation shows important aspects of the data. |
| **Develop and evaluate inferences and predictions that are based on data** | • propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions. |
| **Understand and apply basic concepts of probability** | • describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible;  
• predict the probability of outcomes of simple experiments and test the predictions;  
• understand that the measure of the likelihood of an event can be represented by a number from 0 to 1. |
Problem Solving Standard for Grades 3-5

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard for Grades 3-5

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Communication Standard for Grades 3-5

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.
## Connections Standard for Grades 3-5

*Instructional programs from prekindergarten through grade 12 should enable all students to—*

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

---

## Representation Standard for Grades 3-5

*Instructional programs from prekindergarten through grade 12 should enable all students to—*

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.
Overview

Standards for Grades 3-5

Students enter grade 3 with an interest in learning mathematics. Nearly three-quarters of U.S. fourth graders report liking mathematics, seeing it as practical and important. If mathematics continues to be seen as interesting and understandable, students will remain engaged. If learning becomes simply a process of mimicking and memorizing, students' interest is likely to diminish.

Interwoven through the Content Standards for grades 3–5 are three crucial mathematical themes—multiplicative thinking, equivalence, and computational fluency. The focus on multiplicative reasoning develops knowledge that students build on as they move into the middle grades, where the emphasis is on proportional reasoning. As a part of multiplicative reasoning, students in grades 3–5 should build their understanding of fractions as a part of a whole and as division.

The concept of equivalence helps students learn different mathematical representations and offers a way to explore algebraic ideas. Students should develop computational fluency—efficient and accurate methods for computing that are based on well-understood properties and number relationships. For example, 298 × 42 can be thought of as (300 × 42) – (2 × 42), or 41 × 16 can be computed by multiplying 41 × 8 to get 328 and then doubling 328 to get 656. When these three themes are emphasized, the expectations for grades 3–5 reinforce two major objectives of mathematics learning: making sense of mathematical ideas and acquiring the skills and understandings needed to solve problems.

In grades 3–5, algebraic ideas emerge and are investigated by children. For example, students in these grades are able to make a general statement about how one variable is related to another variable. If a sandwich costs $3, you can figure out how many dollars any number of sandwiches cost by multiplying that number by 3. In this case, students have developed a model of a proportional relationship: the value of one variable is always 3 times the value of the other, or \( C = 3 \times n \).

Given their central role in shaping the mathematics learning of students in these grades, teachers must recognize the need to develop mathematical expertise. Some elementary schools identify a "mathematics teacher-leader," who can support other teachers in their instruction and professional development. Other schools use "mathematics specialists" at the upper elementary grade levels, who assume primary responsibility for teaching mathematics to larger groups of students. Each of these models needs to be explored to enhance the mathematics education of students in grades 3–5.