

NCTM Standards (2000) for Grades 6th through 8th

Number and Operations Standard for Grades 6-8

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	<ul style="list-style-type: none">• work flexibly with fractions, decimals, and percents to solve problems;• compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line;• develop meaning for percents greater than 100 and less than 1;• understand and use ratios and proportions to represent quantitative relationships;• develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation;• use factors, multiples, prime factorization, and relatively prime numbers to solve problems;• develop meaning for integers and represent and compare quantities with them.
Understand meanings of operations and how they relate to one another	<ul style="list-style-type: none">• understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;• use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals;• understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.
Compute fluently and make reasonable estimates	<ul style="list-style-type: none">• select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods;• develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use;• develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results;• develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.

Algebra Standard for Grades 6-8

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Understand patterns, relations, and functions	<ul style="list-style-type: none">• represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;• relate and compare different forms of representation for a relationship;• identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.
Represent and analyze mathematical situations and structures using algebraic symbols	<ul style="list-style-type: none">• develop an initial conceptual understanding of different uses of variables;• explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope;• use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships;• recognize and generate equivalent forms for simple algebraic expressions and solve linear equations
Use mathematical models to represent and understand quantitative relationships	<ul style="list-style-type: none">• model and solve contextualized problems using various representations, such as graphs, tables, and equations.
Analyze change in various contexts	<ul style="list-style-type: none">• use graphs to analyze the nature of changes in quantities in linear relationships.

Geometry Standard for Grades 6-8

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	<ul style="list-style-type: none"> • precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; • understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; • create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	<ul style="list-style-type: none"> • use coordinate geometry to represent and examine the properties of geometric shapes; • use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.
Apply transformations and use symmetry to analyze mathematical situations	<ul style="list-style-type: none"> • describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling; • examine the congruence, similarity, and line or rotational symmetry of objects using transformations.
Use visualization, spatial reasoning, and geometric modeling to solve problems	<ul style="list-style-type: none"> • draw geometric objects with specified properties, such as side lengths or angle measures; • use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; • use visual tools such as networks to represent and solve problems; • use geometric models to represent and explain numerical and algebraic relationships; • recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.

Measurement Standard for Grades 6-8

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Understand measurable attributes of objects and the units, systems, and processes of measurement	<ul style="list-style-type: none">• understand both metric and customary systems of measurement;• understand relationships among units and convert from one unit to another within the same system;• understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.
Apply appropriate techniques, tools, and formulas to determine measurements	<ul style="list-style-type: none">• use common benchmarks to select appropriate methods for estimating measurements;• select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision;• develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes;• develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders;• solve problems involving scale factors, using ratio and proportion;• solve simple problems involving rates and derived measurements for such attributes as velocity and density.

Data Analysis and Probability Standard for Grades 6-8

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 6–8 all students should—
Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them	<ul style="list-style-type: none"> • formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population; • select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots.
Select and use appropriate statistical methods to analyze data	<ul style="list-style-type: none"> • find, use, and interpret measures of center and spread, including mean and interquartile range; • discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatterplots.
Develop and evaluate inferences and predictions that are based on data	<ul style="list-style-type: none"> • use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken; • make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit; • use conjectures to formulate new questions and plan new studies to answer them.
Understand and apply basic concepts of probability	<ul style="list-style-type: none"> • understand and use appropriate terminology to describe complementary and mutually exclusive events; • use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations; • compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

Problem Solving Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation Standard for Grades 6-8

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Overview

Standards for Grades 6-8

The middle grades represent a significant turning point in students' lives. During the middle grades, students solidify conceptions about themselves as learners of mathematics. They arrive at conclusions about their competence in mathematics, their attitudes, their interest, and their motivation. These conceptions will influence how they approach the study of mathematics in later years, which in turn will affect their later career and personal opportunities.



If middle school students find both challenge and support in their mathematics classes, they will be drawn to the subject. They will be able to use their emerging capabilities of finding and imposing structure, conjecturing and verifying, thinking hypothetically, comprehending cause and effect, and engaging in abstraction and generalization.

As in all the grade bands, students in the middle grades need a balanced mathematics program that encompasses all ten Standards, including significant amounts of algebra and geometry. Algebra and geometry are crucial to success in the later study of mathematics and also in many situations that arise outside the mathematics classroom. Students should see that these subjects are interconnected with each other and with other content areas in the curriculum. For example, students might be asked to explain the number of tiles that will be needed to make borders around pools of various lengths and widths, as in figure 2. Students might develop various formulas to express this relationship on the basis of a table or their reasoning about the situation; for example, "You need $L + 2$ tiles across the top and the same number across the bottom. And you need W tiles on the left and the right. So all together, the number of tiles needed is $T = 2(L + 2) + 2W$."

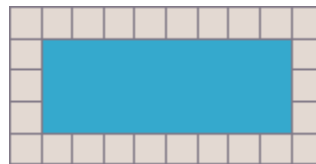


Fig. 2. The "swimming pool" problem

Students' understanding of these crucial ideas should be developed over all three years in the middle grades and across a broad range of mathematics content. This approach is a challenging alternative to the practice of offering a select group of middle-grades students a one-year course that focuses narrowly on algebra or geometry. However, all middle-grades students will benefit from a rich and integrated treatment of mathematics content.

By the end of the eighth grade, students should have a solid background in algebra and other areas that will prepare them to enter substantive high school courses.

Middle-grades mathematics also needs to prepare students to deal with quantitative situations in their lives outside school. For example, consumer magazines regularly publish comparisons of characteristics of various consumer products, such as the quality of peanut butter, the duration of rechargeable batteries, or the cost, size, and gas mileage of automobiles. When using data from such sources, students need to determine which data are appropriate for their needs, to understand how the data were gathered at the source, and to consider limitations that could affect interpretation.

Special attention must be given to the preparation and ongoing professional support of middle-grades teachers. They need a deep understanding of mathematical ideas, pedagogical practices, interdisciplinary teaching approaches, how students learn mathematics, and adolescent development. States and provinces need to give much more attention to the development of special preparation programs for teachers of mathematics in the middle grades.