

NCTM Standards (2000) for Grades 9th through 12th

Number and Operations Standard for Grades 9-12

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 9–12 all students should—
Understand numbers, ways of representing numbers, relationships among numbers, and number systems	<ul style="list-style-type: none">• develop a deeper understanding of very large and very small numbers and of various representations of them;• compare and contrast the properties of numbers and number systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions;• understand vectors and matrices as systems that have some of the properties of the real-number system;• use number-theory arguments to justify relationships involving whole numbers.
Understand meanings of operations and how they relate to one another	<ul style="list-style-type: none">• judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities;• develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices;• develop an understanding of permutations and combinations as counting techniques.
Compute fluently and make reasonable estimates	<ul style="list-style-type: none">• develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.• judge the reasonableness of numerical computations and their results.

Algebra Standard for Grades 9-12

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 9–12 all students should—
Understand patterns, relations, and functions	<ul style="list-style-type: none"> • generalize patterns using explicitly defined and recursively defined functions; • understand relations and functions and select, convert flexibly among, and use various representations for them; • analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior; • understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions; • understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions; • interpret representations of functions of two variables
Represent and analyze mathematical situations and structures using algebraic symbols	<ul style="list-style-type: none"> • understand the meaning of equivalent forms of expressions, equations, inequalities, and relations; • write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases; • use symbolic algebra to represent and explain mathematical relationships; • use a variety of symbolic representations, including recursive and parametric equations, for functions and relations; • judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology.
Use mathematical models to represent and understand quantitative relationships	<ul style="list-style-type: none"> • identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships; • use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts; • draw reasonable conclusions about a situation being modeled.
Analyze change in various contexts	<ul style="list-style-type: none"> • approximate and interpret rates of change from graphical and numerical data

Geometry Standard for Grades 9-12

Expectations

Instructional programs from prekindergarten through grade 12 should enable all students to—	In grades 9–12 all students should—
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships	<ul style="list-style-type: none"> • analyze properties and determine attributes of two- and three-dimensional objects; • explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them; • establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others; • use trigonometric relationships to determine lengths and angle measures.
Specify locations and describe spatial relationships using coordinate geometry and other representational systems	<ul style="list-style-type: none"> • use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations; • investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
Apply transformations and use symmetry to analyze mathematical situations	<ul style="list-style-type: none"> • understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices; • use various representations to help understand the effects of simple transformations and their compositions.
Use visualization, spatial reasoning, and geometric modeling to solve problems	<ul style="list-style-type: none"> • draw and construct representations of two- and three-dimensional geometric objects using a variety of tools; • visualize three-dimensional objects and spaces from different perspectives and analyze their cross sections; • use vertex-edge graphs to model and solve problems; • use geometric models to gain insights into, and answer questions in, other areas of mathematics; • use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

Measurement Standard for Grades 9-12

Expectations

<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p>	<p>In grades 9–12 all students should—</p>
<p>Understand measurable attributes of objects and the units, systems, and processes of measurement</p>	<ul style="list-style-type: none"> • make decisions about units and scales that are appropriate for problem situations involving measurement.
<p>Apply appropriate techniques, tools, and formulas to determine measurements</p>	<ul style="list-style-type: none"> • analyze precision, accuracy, and approximate error in measurement situations; • understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders; • apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations; • use unit analysis to check measurement computations.

Data Analysis and Probability Standard for Grades 9-12

Expectations

<p>Instructional programs from prekindergarten through grade 12 should enable all students to—</p>	<p>In grades 9–12 all students should—</p>
<p>Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them</p>	<ul style="list-style-type: none"> • understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each; • know the characteristics of well-designed studies, including the role of randomization in surveys and experiments; • understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable; • understand histograms, parallel box plots, and scatterplots and use them to display data; • compute basic statistics and understand the distinction between a statistic and a parameter.

<p>Select and use appropriate statistical methods to analyze data</p>	<ul style="list-style-type: none"> • for univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics; • for bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools; • display and discuss bivariate data where at least one variable is categorical; • recognize how linear transformations of univariate data affect shape, center, and spread; • identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.
<p>Develop and evaluate inferences and predictions that are based on data</p>	<ul style="list-style-type: none"> • use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions; • understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference; • evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions; • understand how basic statistical techniques are used to monitor process characteristics in the workplace.
<p>Understand and apply basic concepts of probability</p>	<ul style="list-style-type: none"> • understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases; • use simulations to construct empirical probability distributions; • compute and interpret the expected value of random variables in simple cases; • understand the concepts of conditional probability and independent events; • understand how to compute the probability of a compound event.

Problem Solving Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation Standard for Grades 9-12

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Overview

In secondary school, all students should learn an ambitious common foundation of mathematical ideas and applications. This shared mathematical understanding is as important for students who will enter the workplace as it is for those who will pursue further study in mathematics and science. All students should study mathematics in each of the four years that they are enrolled in high school.



Because students' interests and aspirations may change during and after high school, their mathematics education should guarantee access to a broad spectrum of career and educational options. They should experience the interplay of algebra, geometry, statistics, probability, and discrete mathematics. They need to understand the fundamental mathematical concepts of function and relation, invariance, and transformation. They should be adept at visualizing, describing, and analyzing situations in mathematical terms. And they need to be able to justify and prove mathematically based ideas.

High school mathematics builds on the skills and understandings developed in the lower grades. For example, students should enter high school with extensive experience in modeling various patterns and relationships. High school students might explore the following problem:

A student strained her knee in an intramural volleyball game, and her doctor prescribed an anti-inflammatory drug to reduce the swelling. She is to take two 220-milligram tablets every 8 hours for 10 days. If her kidneys filtered 60% of this drug from her body every 8 hours, how much of the drug was in her system after 10 days? How much of the drug would have been in her system if she had continued to take the drug for a year?

(See next page...)

	A	B
1	440	
2	616	
3	695.4	
4	714.96	
5	726.924	
6	732.2856	
7	734.5804	
8	735.68376	
9	736.49294	
10	737.084376	
11	737.504764	
12	737.7908	
13	738.00412	
14	738.163568	
15	738.284488	
16	738.369464	
17	738.420752	
18	738.45088	
19	738.46992	
20	738.47776	
21	733.3333301	
22	733.333332	
23	733.3333326	
24	733.3333331	
25		

Fig. 3. A spreadsheet computation of the "drug dosage" problem

Students might represent the equation informally as $\text{NEXT} = 0.4(\text{NOW}) + 440$, start at 440. Entering this relationship in a spreadsheet (see fig. 3), they could note that an "equilibrium" value of about $733 \frac{1}{3}$ milligrams is reached. This investigation might lead to explorations of finite sequences and series.

High school students can study mathematics that extends beyond the material expected of all students in at least three ways. One is to include in the curriculum material that extends the foundational material in depth or sophistication. Two other approaches make use of supplementary courses. In the first, students enroll in additional courses concurrent with those expected of all students. In the second, students complete a three-year version of the shared material and then take other mathematics courses. In both situations, students can choose from such courses as computer science, technical mathematics, statistics, and calculus. Each of these approaches has the essential property that all students learn the same foundation of mathematics but some, if they wish, can study additional mathematics.

The Standards for high school students are ambitious. The demands made on high school teachers in achieving the Standards will require extended and sustained professional development and a large degree of administrative support.