### NCTM Standards (2000) for Grades Pre-K through 2nd

#### Number and Operations Standard for Grades Pre-K-2

<table>
<thead>
<tr>
<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
<th>In prekindergarten through grade 2 all students should—</th>
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| Understand numbers, ways of representing numbers, relationships among numbers, and number systems | • count with understanding and recognize "how many" in sets of objects;  
 • use multiple models to develop initial understandings of place value and the base-ten number system;  
 • develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections;  
 • develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers;  
 • connect number words and numerals to the quantities they represent, using various physical models and representations;  
 • understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2. |
| Understand meanings of operations and how they relate to one another | • understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations;  
 • understand the effects of adding and subtracting whole numbers;  
 • understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally. |
| Compute fluently and make reasonable estimates | • develop and use strategies for whole-number computations, with a focus on addition and subtraction;  
 • develop fluency with basic number combinations for addition and subtraction;  
 • use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators. |
## Algebra Standard for Grades Pre-K-2

### Expectations

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<th>Instructional programs from prekindergarten through grade 12 should enable all students to—</th>
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| **Understand patterns, relations, and functions** | • sort, classify, and order objects by size, number, and other properties;  
• recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another;  
• analyze how both repeating and growing patterns are generated. |
| **Represent and analyze mathematical situations and structures using algebraic symbols** | • illustrate general principles and properties of operations, such as commutativity, using specific numbers;  
• use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations. |
| **Use mathematical models to represent and understand quantitative relationships** | • model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols. |
| **Analyze change in various contexts** | • describe qualitative change, such as a student's growing taller;  
• describe quantitative change, such as a student's growing two inches in one year. |
# Geometry Standard for Grades Pre-K-2

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| **Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships** | • recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;  
• describe attributes and parts of two- and three-dimensional shapes;  
• investigate and predict the results of putting together and taking apart two- and three-dimensional shapes. |
| **Specify locations and describe spatial relationships using coordinate geometry and other representational systems** | • describe, name, and interpret relative positions in space and apply ideas about relative position;  
• describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance;  
• find and name locations with simple relationships such as "near to" and in coordinate systems such as maps. |
| **Apply transformations and use symmetry to analyze mathematical situations** | • recognize and apply slides, flips, and turns;  
• recognize and create shapes that have symmetry. |
| **Use visualization, spatial reasoning, and geometric modeling to solve problems** | • create mental images of geometric shapes using spatial memory and spatial visualization;  
• recognize and represent shapes from different perspectives;  
• relate ideas in geometry to ideas in number and measurement;  
• recognize geometric shapes and structures in the environment and specify their location. |
# Measurement Standard for Grades Pre-K-2

## Expectations

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| **Understand measurable attributes of objects and the units, systems, and processes of measurement** | • recognize the attributes of length, volume, weight, area, and time;  
• compare and order objects according to these attributes;  
• understand how to measure using nonstandard and standard units;  
• select an appropriate unit and tool for the attribute being measured. |
| **Apply appropriate techniques, tools, and formulas to determine measurements** | • measure with multiple copies of units of the same size, such as paper clips laid end to end;  
• use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meterstick;  
• use tools to measure;  
• develop common referents for measures to make comparisons and estimates. |
## Data Analysis and Probability Standard for Grades Pre-K-2

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| Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them | • pose questions and gather data about themselves and their surroundings;  
• sort and classify objects according to their attributes and organize data about the objects;  
• represent data using concrete objects, pictures, and graphs. |
| Select and use appropriate statistical methods to analyze data | • describe parts of the data and the set of data as a whole to determine what the data show. |
| Develop and evaluate inferences and predictions that are based on data | • discuss events related to students' experiences as likely or unlikely. |
| Understand and apply basic concepts of probability | |

## Problem Solving Standard for Grades Pre-K-2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;  
- solve problems that arise in mathematics and in other contexts;  
- apply and adapt a variety of appropriate strategies to solve problems;  
- monitor and reflect on the process of mathematical problem solving.
Reasoning and Proof Standard for Grades Pre-K-2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication Standard for Grades Pre-K-2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections Standard for Grades Pre-K-2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.
### Representation Standard for Grades Pre-K-2

Instructional programs from prekindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.
Overview

Standards for Grades Pre-K-2

Developing a solid mathematical foundation from prekindergarten through second grade is essential for every child. In these grades, students are building beliefs about what mathematics is, about what it means to know and do mathematics, and about themselves as mathematics learners. These beliefs influence their thinking about, performance in, and attitudes toward, mathematics and decisions related to studying mathematics in later years.

Children develop many mathematical concepts, at least in their intuitive beginnings, even before they reach school age. Infants spontaneously recognize and discriminate among small numbers of objects, and many preschool children possess a substantial body of informal mathematical knowledge. Adults can foster children's mathematical development from the youngest ages by providing environments rich in language and where thinking is encouraged, uniqueness is valued, and exploration is supported.

Children are likely to enter formal school settings with different levels of mathematics understanding, reflecting their opportunity to have learned mathematics. Some children will need additional support so that they do not start school at a disadvantage. Early assessments should be used not to sort children but to gain information for teaching and for potential early interventions.

All students deserve high-quality programs that include significant mathematics presented in a manner that respects both the mathematics and the nature of young children. These programs must build on and extend students' intuitive and informal mathematical knowledge. They must be grounded in a knowledge of child development and provide environments that encourage students to be active learners and accept new challenges. They need to develop a strong conceptual framework while encouraging and developing students' skills and their natural inclination to solve problems.

At the core of mathematics programs in prekindergarten through grade 2 are the Number and Operations and Geometry Standards. For example, it is absolutely essential that students develop a solid understanding of the base-ten numeration system in prekindergarten through grade 2. They must recognize that the word ten may represent a single entity (1 ten) or ten separate units (10 ones) and that these representations are interchangeable. Using concrete materials and calculators in appropriate ways can help students learn these concepts.

Understandings of patterns, measurement, and data contribute to the understanding of number and geometry and are learned in conjunction with them. Similarly, the Process Standards of Problem Solving, Reasoning and Proof, Communication, Connections, and Representation both
support and augment the Content Standards. Even at this age, guided work with calculators can enable students to explore number and patterns, focus on problem-solving processes, and investigate realistic applications. See, for example, the problem in figure 1.

**Fig. 1.** A calculator activity to help develop understanding of place value

<table>
<thead>
<tr>
<th>Make a New Number</th>
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<tbody>
<tr>
<td>Use a calculator. Start with 78. Do not press clear. Make the next number:</td>
</tr>
<tr>
<td>98</td>
</tr>
<tr>
<td>48</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>11 R</td>
</tr>
<tr>
<td>119</td>
</tr>
</tbody>
</table>

In the elementary grades, it often happens that specific blocks of time are not allotted to instruction in particular subjects. It is essential for students in the elementary grades to study mathematics for an hour a day under the guidance of teachers who enjoy mathematics and are prepared to teach it well. This basic requirement takes thoughtful arrangements of scheduling and staffing—whether by shared teaching responsibilities, the use of mathematics specialists, or other creative administrative means.